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Space Encounter Modeling Study
(Mathematical Models)

by

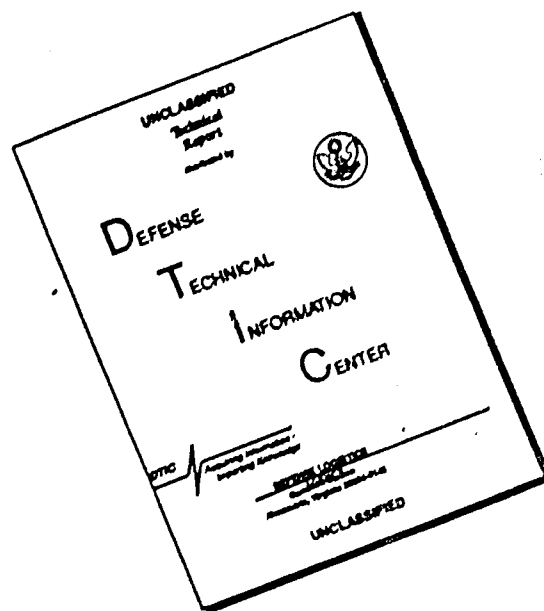Michael Liveright

IIT Research Institute
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AIR FORCE ARMAMENT LABORATORY

RESEARCH AND TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

EGLIN AIR FORCE BASE, FLORIDA

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SPACE ENCOUNTER MODELING STUDY
(Mathematical Models)

by

Michael Liveright
IIT Research Institute

FOREWORD

This final report (Unclassified Title) "Space Encounter Modeling Study" describes research conducted by the IIT Research Institute under Contract AF08(635)-4203, IITRI Program K6064. The work period was 28 Feb 1964 to 28 Feb 1965. The program was administered by the Air Force Armament Laboratory (ATWR), Eglin Air Force Base, Florida. Mr. R. L. Hill, Jr. was the AFATL Project Officer.

This report comprises four volumes:

     I Model Formulation (Unclassified)

     II Mathematical Models (Unclassified)

     III Sample Problems (Secret)

     IV Computer Program (Unclassified)

The IITRI Project team included L. A. C. Barbarek (Project Engineer), M. Liveright(Statistician and Programmer), E. P. Bergmann (Warhead Evaluation) and T. Martin (Fire Control).

Information in this report is embargoed under the Deparment of State International Traffic In Arms Regulations. This report may be released to foreign governments by departments or agencies of the U. S. Government subject to approval of the Air Force Armament Laboratory (ATWR), Eglin Air Force Base, Florida, or higher authority within the Department of the Air Force. Private individuals or firms require a Department of State export license.

This report has been reviewed and is approved.

DAVID K. DEAN, Colonel, USAF
Chief, Weapons Division

ABSTRACT

Studies conducted to provide the Air Force with mathematical models of probable space tactical situations are described in this four-volume report. The need for these models was established in order that the Air Force have means of guiding the logical and timely development and evaluation of nonnuclear space warhead concepts.

The model described can determine:

- Warhead fuzing and/or firing time (projected warheads) for a constant velocity intercept with a given miss-distance or miss-distance distribution.

- Warhead-target intercept error dispersion as a function of warhead fuzing and/or firing time for any combination of intercept parameter errors.

- Probability of hit of a given size vulnerable volume by a uniform random pattern-type warhead.

- Over-all probability of hit that is optimum for a given miss-distance distribution.

- Over-all probability of hit using a fuzing logic based upon a given value of some intercept parameters (i.e., line-of-sight range, line-of-sight bearing rate).

CONTENTS

ILLUSTRATIONS

ONE

INTERCEPT GEOMETRY MATHEMATICS

The first problem in evaluating a delivery-warhead
system is to compute the position, velocities, and
angles at the ideal intercept point from the initial
launch parameters. The FLYCUM subroutine performs this
computation.

Figure 1 shows the angles and distances that must
be considered in this problem. The initial parameters
are denoted by a 1 subscript, whereas the intercept
parameters are denoted by a 2 subscript.

The given parameters are:

R_1 initial range of the target
B_1 target bearings
E_1 target elevation
V_{targ} target speed
R_{1a} line-of-sight to velocity angle
B_{1a} horizontal to velocity angle in the (i,p)
     or reduction plane
V_{part} the speed of the delivery vehicle, or
     particle cloud.

The output parameters that must be computed are:

$\dot{R}_1$ initial range rate
$\dot{A}_1$ initial angle (sweep) rate
T_{flit} time of flight to the intercept point
R_2 target's intercept range
B_2 bearing
E_2 elevation
R_{2a} line-of-sight to velocity angle
B_{2a} horizontal to velocity angle in the
     reduction plane
$\dot{R}_2$ intercept range rate
$\dot{A}_2$ intercept angle rate
D_a angle between the two lines of sight.

Using the cosine law, the intercept geometry can be
expressed by the following equation which may be
solved for T_{flit}, the time-of-flight.

1

$$(V_{part} * T_{flit})^2 = (V_{targ} * T_{flit})^2 + (R_1)^2 \quad (1)$$

$$+ 2*R_1 * T_{flit} * V_{targ} \cos (R_{1a})$$

In the situations where this equation has two solutions, the minimum nonnegative solution is chosen. If no non-negative solutions occur, then there can be no inter-cept and the problem is terminated.

After the flight time has been established, it is necessary to determined the position of the target at intercept. It is useful to consider a coordinate system (X,Y,Z) such that the X and Y axes lie in the horizontal plane and the X axis is along the motion of the initial line-of-sight in the horizontal plane. In this system, the target coordinates are:

$$
\begin{aligned}
X &= R_1 * \cos (E_1), \\
Y &= 0, \text{ and} \\
Z &= R_1 * \sin (E_1).
\end{aligned}
\quad (2)
$$

Relative to this coordinate system the velocity compo-nents of the target are:

$$\dot{X} = V_{targ} * \begin{pmatrix} \cos (R_{1a}) * \cos (E_1) \\ -\sin (R_{1a}) * \sin (E_1) * \sin (B_{1a}) \end{pmatrix},$$

$$\dot{Y} = V_{targ} * \sin (R_{1a}) * \cos (B_{1a}), \text{ and}$$

$$\dot{Z} = V_{targ} * \begin{pmatrix} \cos (R_{1a}) * \sin (E_1) \\ +\sin (B_{1a}) * \sin (R_{1a}) * \cos (E_1) \end{pmatrix}$$

It is easy enough to determine the target position after a time of T_{flit} and, thus, to determine the intercept position in the R, B, E system as show below.

2

$$R_2 = \left(\begin{array}{l} (X_1 + \dot{X}_1 * T_{flit})^2 + (Y_1 + \dot{Y}_1 * T_{flit})^2 \\ \quad + (Z_1 + \dot{Z}_1 * T_{flit})^2 \end{array} \right)$$

$$B_2 = B_1 + \tan^{-1} \left(\frac{Y_1 + \dot{Y}_1 * T_{flit}}{X_1 + \dot{X}_1 * T_{flit}} \right)$$

$$E_2 = \tan^{-1} \frac{Z_1 + \dot{Z}_1 * T_{flit}}{\sqrt{(X_1 + \dot{X}_1 * T_{flit})^2 + (Y_1 + \dot{Y}_1 * T_{flit})^2}}$$

Once the position of the target at intercept has been determined it is possible to compute the velocity angles relative to this system. Since the velocity components are already expressed in the (X, Y, Z) systems it is just necessary to perform a transformation into the (r_2, i_2, p_2) system. As Figure 1 shows, this is done by a rotation around the Z-axis of $dB = B_2 - B_1$ followed by a rotation around the new Y-axis by E_2. This transformation is expressed in matrix notation by the equations:

$$\begin{pmatrix} \dot{r}_2 \\ \dot{i}_2 \\ \dot{p}_2 \end{pmatrix} = \begin{pmatrix} \cos(E_1), 0, \sin(E_1) \\ 0, 1, 0 \\ -\sin(E_1), 0, \cos(E_1) \end{pmatrix} * \begin{pmatrix} \cos(DB), \sin(DB), 0 \\ -\sin(DB), \cos(DB), 0 \\ 0, 0, 1 \end{pmatrix} * \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}$$

The intercept velocity angles may then be determined:

$$R_{2a} = \tan^{-1} \left((\dot{i}_2^2 + \dot{p}_2^2)^{1/2} / \dot{R}_2 \right), \quad B_{2a} = \dot{p}_2 / \dot{R}_2$$

The rates can easily be computed from the data already available.

$$\dot{R}_n = V_{targ} * \cos(R_{na})$$

$$\dot{A}_n = \frac{V_{-targ} * \sin(R_{na})}{R_n}$$

where $n = 1$ or 2.

3
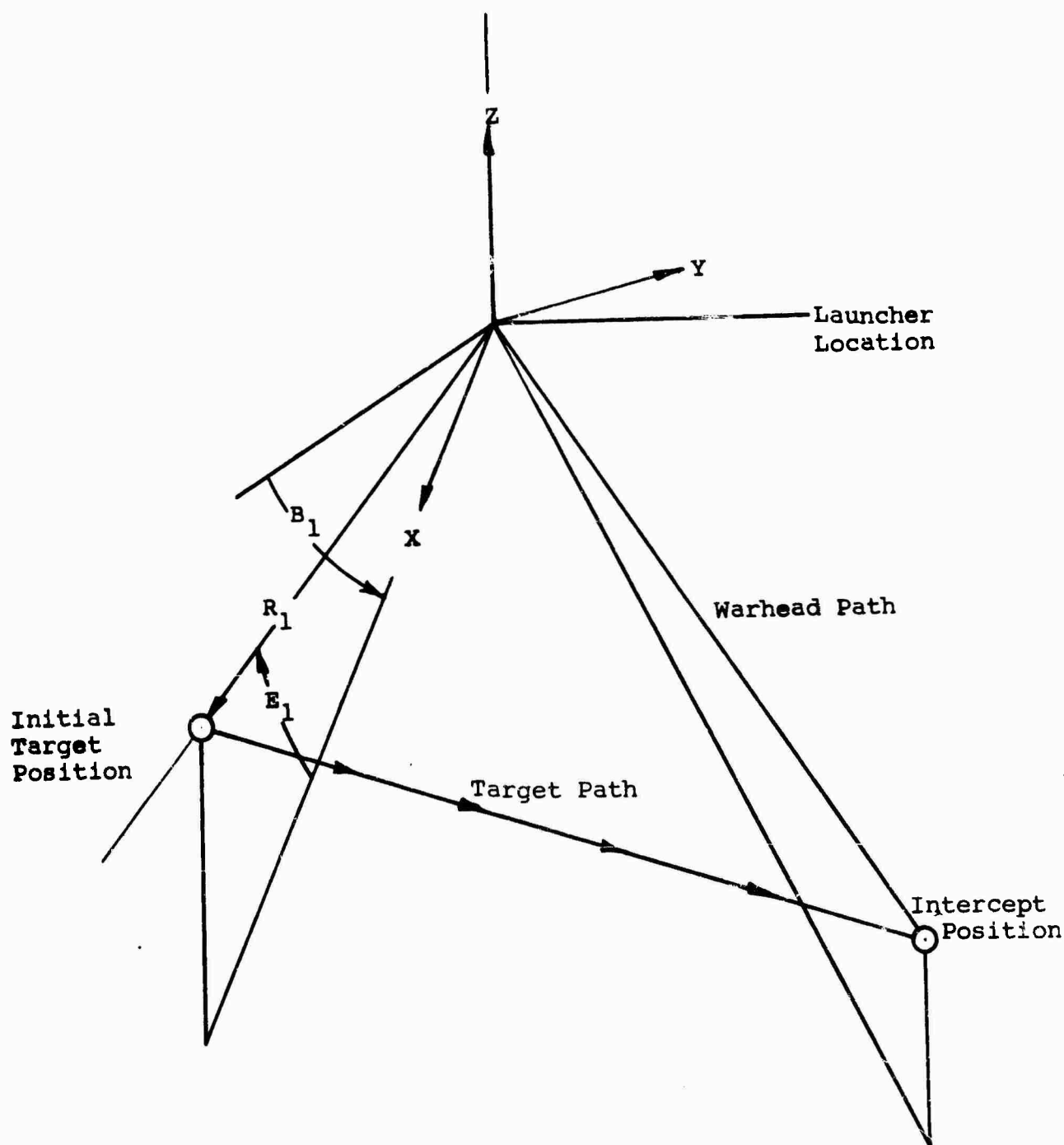
(a) At initial target position

Figure 1 Angles and distances in target flight
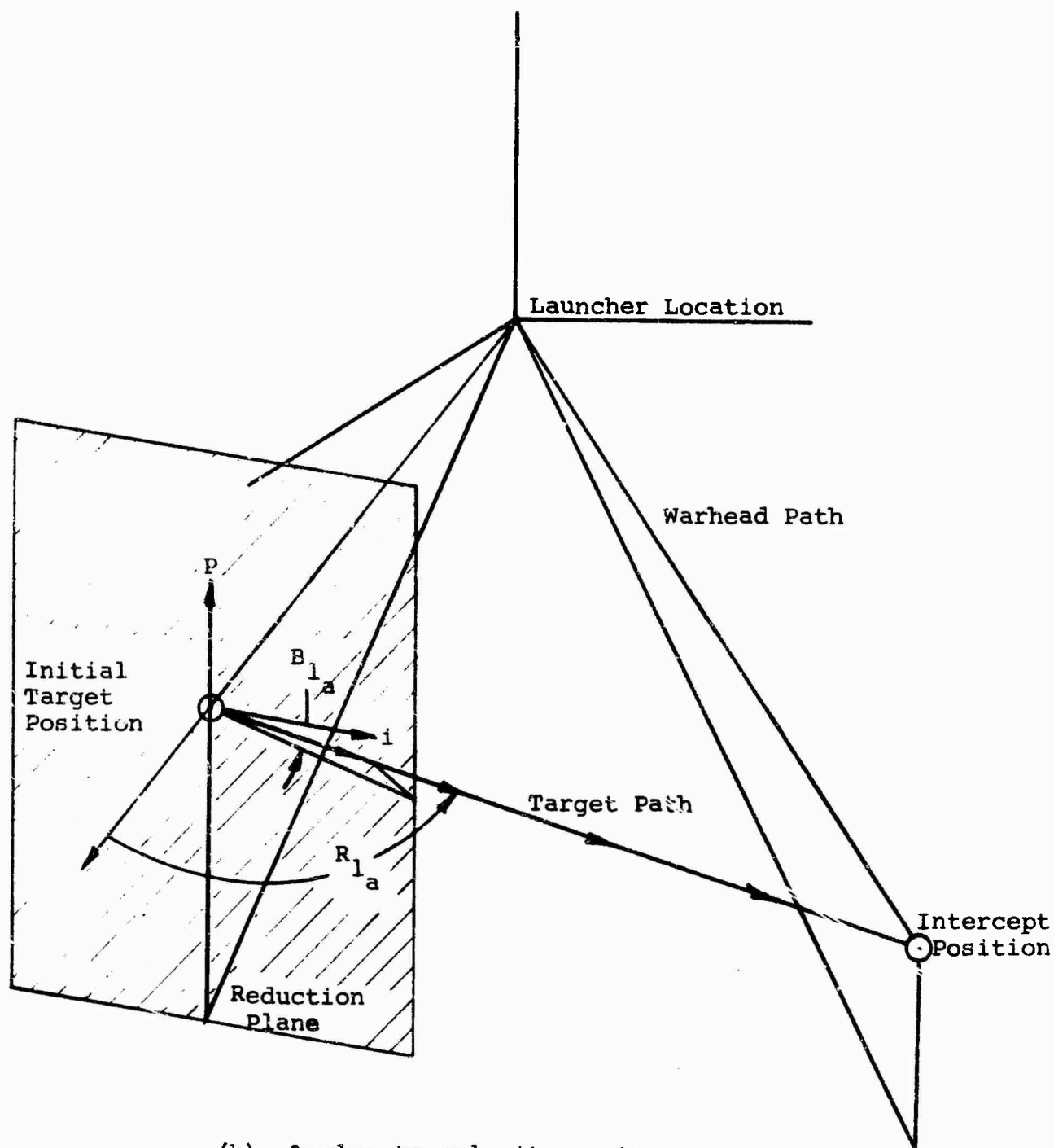
(b)   Angles to velocity vector

Figure 1 (Cont.)   Angles and distances in target flight

Launcher Location

Warhead Path

B_2

D_a

Initial
Target
Position

R_2

Target Path

E_2

Intercept
Position

R_{2_a}

(c)   At intercept position

Figure 1 (Cont.)   Angles and distances in target flight

Launcher Location

Warhead Path =
$T_{flit}{}^* V_{part}$

R_1

Initial
Target
Position

Target Path =
$T_{flit}{}^* V_{targ}$

R_{1_a}
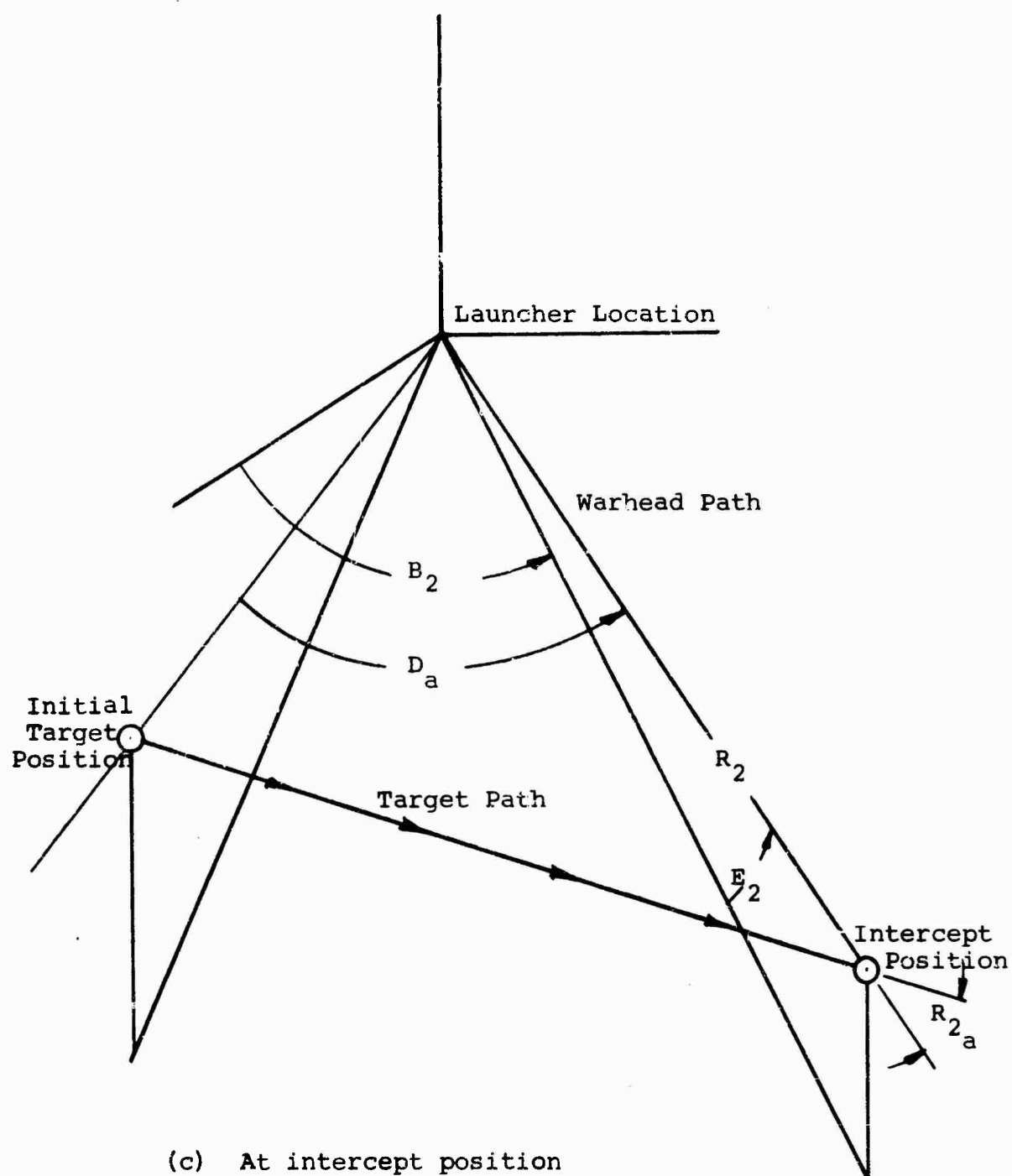
(d)   To compute T_{flit}

Figure 1 (Concluded)   Angles and distances in target
flight

7

Finally, the angle between the two lines-of-sight can be computed using the cosine formula:

$$D_a = \tan^{-1}\left(\frac{V_{targ} * \sin(R_{la})}{R_l/T_{flit} + V_{targ} * \cos(R_{la})}\right)$$

Thus, all the required intercept parameters may be computed.
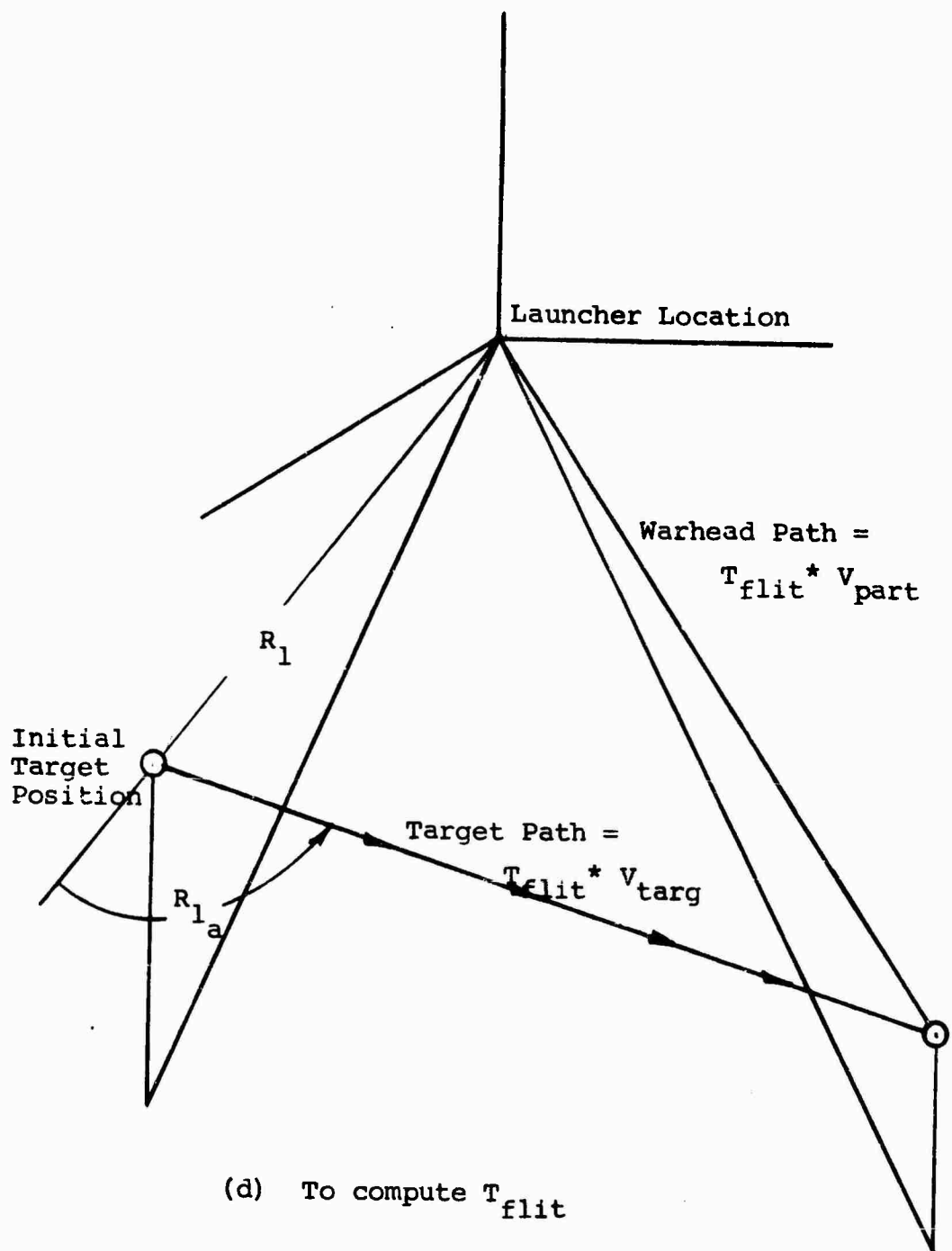
ERROR DISTRIBUTION MATHEMATICS

After computing the intercept situation, it is
necessary to compute the effect of the various errors
in launching the warhead on the position of the war-
head centroid relative to the target at the nominal
intercept time. The ERRCM and INDVAR subroutines are
the programs that generate the distribution of the
centroid position.

Three general classes of errors may affect the
position of the warhead centroid. First, there may be
errors in measuring the position or velocity of the
target by the warhead launching system. These errors
cause the warhead to be launched along a path different
from the ideal path, and thus contribute to the possi-
bility of missing the target. Second, there may be
errors in launching the warhead along its flight path.
These also contribute to missing the target. Third,
there may be errors in the position of a specific par-
ticle relative to the flight path of the delivery
vehicle.

Any single error (if it is reasonably small com-
pared with the intercept parameters) will cause an
error in the position of the centroid of the particles
of the warhead. This error will lie along a single
line. For example, an error in the launch bearing will
cause an error in the centroid that is horizontal and
is perpendicular to the line-of-sight to the target at
nominal intercept time. If the error is normally dis-
tributed, then its effect will be a normal distribution
of centroid positions lying along this same line. The
other errors lead to the same type of linear normal
distributions. If the various errors are all independent,
then they may be combined by expressing them all in a
single coordinate system and adding their respective
biases, variances and covariances. The results of this
combination is a three-dimensional normal distribution.

The errors in measuring the target position may
arise from a number of sources. These errors lie along
the range, bearing elevation (R, B, E) system or the

line-of-sight (r, i, p) system at the time when the
warhead is launched (Figure 2). In the case of these
errors in measuring the target position the line-of-
sight system is the reduction plane system.

A target error may have one of three different
effects, and errors must be provided for each of these
effects. First, it may affect the measurement of range,
bearing, or elevation, or the measurement of the rate
of any of these variables. These errors are denoted by
the following symbols:

Measured range error	$= R_{le}$
Measured range rate error	$= \dot{R}_{le}$
Measured bearing error	$= B_{le}$
Measured bearing rate error	$= \dot{B}_{le}$
Measured elevation error	$= E_{le}$
Measured elevation rate error	$= \dot{E}_{le}$

Second, the error may affect the position measurement
in such a way that an error occurs in a random direction
of the (i,p) plane. This will cause an angular error
in the (i,p) plane denoted by:

$$\text{Measured angular error} \qquad = \lambda_{le}.$$

Third, the error may cause an error in the i or p
direction that is measured in linear units, e.g., feet.
These types of errors are denoted by the symbols:

Measured	i	linear error	$= i_{le}$
Measured	i	linear rate error	$= \dot{i}_{le}$
Measured	p	linear error	$= p_{le}$
Measured	p	linear rate error	$= \dot{p}_{le}.$

For this analysis, we assume that these errors are
all normally distributed, and independent. The bias of
any error A is represented by B(A) and its variance
by V(A).

The following equations show the combined effect
of all of the target errors mentioned previously.

$$B(r_1) = B(R_{le}) + T_{int} * B(\dot{R}_{le})$$

$$B(i_1) = B(\dot{i}_{le}) + R * \cos(E) * B(B_{le}) + T_{int} * (B(\dot{i}_{le})$$
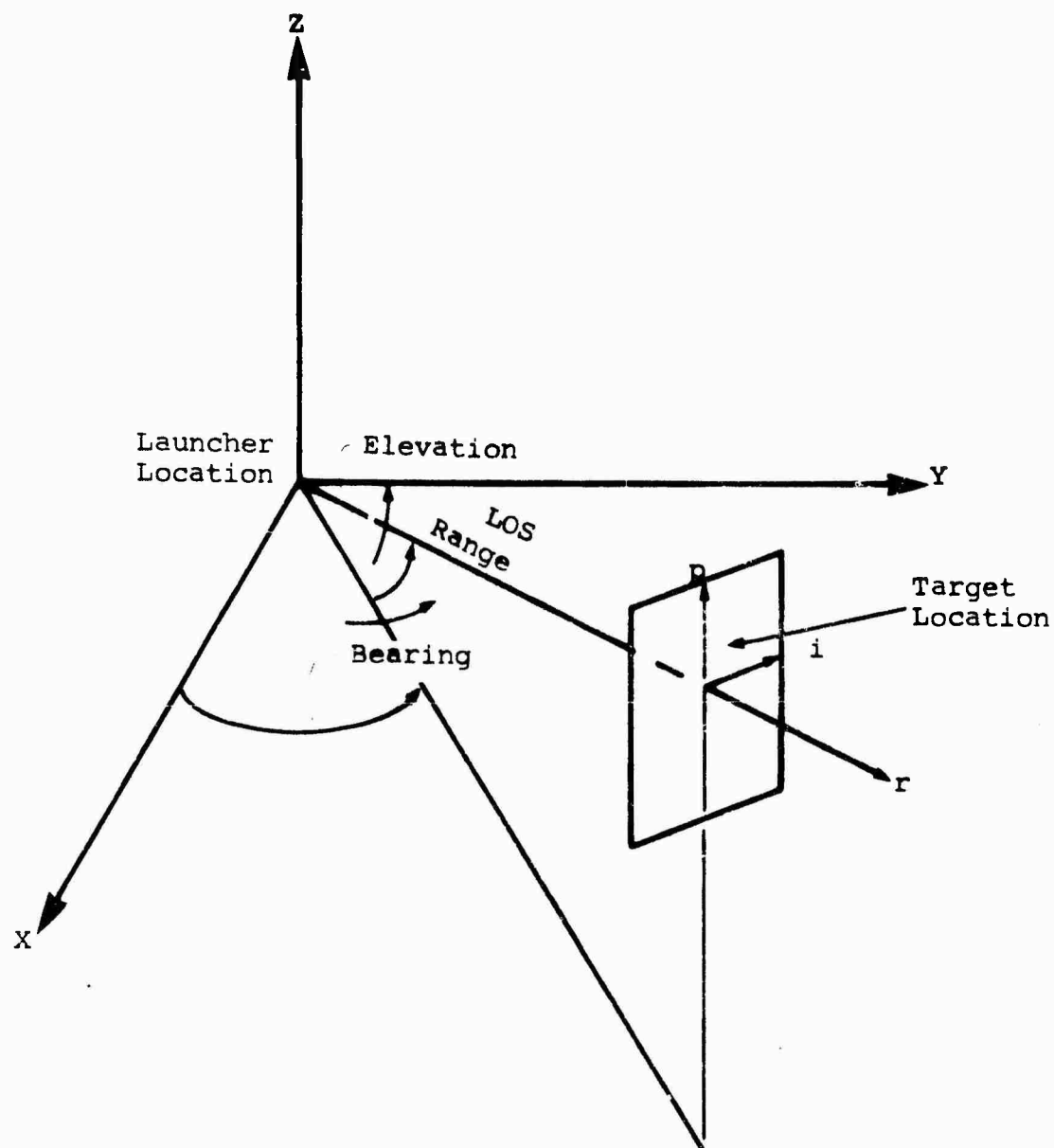
$$+ R * \cos(E) B(B_{le}))$$

10

Figure 2   Relative position of target

11

$$B(p_1) = B(p_{1e}) + R * B(E_{1e}) + T_{int} * (B(\dot{p}_{1e})$$
$$+ R * B(\dot{E}_{1e}))$$

$$V(r_1) = V(R_{1e}) + T_{int}^2 * V(\dot{R}_{1e})$$

$$B(i_1) = V(i_{1e}) = (R * \cos(E))^2 * V(B_{1e})$$
$$+ T_{int}^2 * (V(\dot{i}_{1e}) + (R * \cos(E))^2 * V(\dot{B}_{1e}))$$
$$+ R^2 * V(\lambda_{1e})$$

$$V(p_1) = V(p_{1e}) + R^2 * V(E_{1e}) + T_{int}^2 * (V(\dot{p}_{1e})$$
$$+ R^2 * V(\dot{E}_{1e})) + R^2 * V(\lambda_{1e}).$$

The range R and elevation E in these equations are evaluated at the aiming time or time of fire, and T_{int} is the time between firing the warhead and its intercept with the target.

The errors in launching the warhead are similar to those in measuring the target position except that they will arise from different causes, and they will occur when the warhead ideally hits the target. These errors are denoted by the symbols:

Launch velocity error $= V_{ce}$
Launch bearing error $= B_{2e}$
Launch bearing rate error $= \dot{B}_{2e}$
Launch elevation error $= E_{2e}$
Launch elevation rate error $= \dot{E}_{2e}$.

As with the previous error type, an error may affect the launch in a random direction leading to an angular error in the (i,p) plane denoted by:

Launch angular error $= \lambda_{ce}$.

Finally, we may be interested in a particle of the warhead that differs from the centroid of the warhead in that it has additional errors denoted as

Particle r velocity error $= \dot{r}_{pe}$
Particle i velocity error $= \dot{i}_{pe}$
Particle p Velocity error $= \dot{p}_{pe}$, and
Particle angular error $= \lambda_{pe}$.

It is possible to combine all the errors that are related to the intercept coordinate system as those discussed previously were combined. The following equations show the combined effect of all the errors in luanching the warhead, and in a particle position relative to the warhead.

$$B(r_2) = T_{int} * (B(V_{ce}) + B(\dot{R}_{pe}))$$

$$B(i_2) = R + \cos(E) * B(B_{2e}) + T_{int} * (B(\dot{i}_{pe})$$
$$+ R * \cos(E) * B(\dot{B}_{2e}))$$

$$B(p_2) = R + B(E_{2e}) + T_{int} * (B(\dot{p}_{pe}) + R * B(\dot{E}_{2e}))$$

$$V(r_2) = T_{int}^2 * (V(V_{ce}) + V(\dot{R}_{pe}))$$

$$V(i_2) = R^2 * (V(\lambda_{ce}) + V(\lambda_{pe}) + \cos(E)^2 * V(B_{2e}))$$
$$+ T_{int}^2 * (V(\dot{i}_{pe}) + R^2 \cos(E)^2 * V(\dot{B}_{2e}))$$

$$V(p_2) = R^2 * (V(\lambda_{ce}) + V(\lambda_{pe}) + V(E_{2e}))$$
$$+ T_{int}^2 * V(\dot{p}_{pe}) + R^2 * V(\dot{E}_{2e}))$$

Here, R and E are evaluated at the intercept point.

At this point there are two sets of errors, one expressed in terms of the launch system and the other in terms of the intercept system. The next problem is to express one (the intercept system errors) in terms of the other (the launch system) so that the two sets of errors may be combined into one distribution.

13

The transformation between the intercept system and the launch system is the product of three rotations.

A = First, a rotation around the i_2-axis by an angle of $-E_2$,

B = Second, a rotation around the new p_2'-axis by an angle of $-dB = -(B_2-B_2)$,

C = Third, a rotation around the new $i_2'' = i_1$ axis by an angle of E_1.

$$
\begin{bmatrix} r_1 \\ i_1 \\ p_o \end{bmatrix} =
\overset{\text{Rotation C}}{\begin{bmatrix} \cos(E_1),0,\sin(E_1) \\ 0,1,0 \\ -\sin(E_1),0,\cos(E_1) \end{bmatrix}}
\times
\overset{\text{Rotation B}}{\begin{bmatrix} \cos(dB),-\sin(dB),0 \\ \sin(dB),\cos(dB),0 \\ 0,0,1 \end{bmatrix}}
$$

$$
\times
\overset{\text{Rotation A}}{\begin{bmatrix} \cos(E_2),0,-\sin(E_2) \\ 0,1,0 \\ \sin(E_2),0,\cos(E_2) \end{bmatrix}}
\times
\begin{bmatrix} r_2 \\ i_2 \\ p_2 \end{bmatrix}
$$

It is possible to combine the three rotations into one transformation F, by performing the multiplication of the three matrices. This leads to the following transformation between (r_2, i_2, p_2) and (r_1, i_2, p_2).

$$
\begin{bmatrix} r_1 \\ i_1 \\ p_1 \end{bmatrix} =
\overset{\text{Transformation F}}{\begin{bmatrix} f_{rr},f_{ri},f_{rp} \\ f_{ir},f_{ii},f_{ip} \\ f_{pr},f_{pi},f_{pp} \end{bmatrix}}
\times
\begin{bmatrix} r_2 \\ i_2 \\ p_2 \end{bmatrix}
$$

These coefficients permit the total bias, $TB(a_1)$, total variance, $Tv(a_1)$, and total covariance, $Tcv(a_1,b_1)$ to be computed for a and b equal to r, i, or p. Thus, the distribution coefficients along each of the launch axes may be computed from the equation:

$$TB(a_1) = B(a_1) + f_{ar} * B(r_2) + f_{ai} * B(i_2)$$

$$+ f_{ap} * B(p_2)$$

14

$$TV(a_1) = V(a_1) + f_{ar}^2 * V(r_2) + f_{ai}^2 * V(i_2)$$
$$+ f_{ap}^2 * V(p_2)$$

$$CV (a_1,b_1) = f_{ar} * f_{br} * V(r_2) + f_{ai} * f_{bi} * V(i_2)$$
$$+ f_{ap} * f_{bp} * V(p_2).$$

The errors in the r_1-direction will not be considered in the cloud models used. These errors only affect the time at which a particle hits the target, and only indirectly affect the hit probability. This indirect effect is due to the expansion of the warhead and, since it is assumed that the expansion speed is small relative to the relative speed of the warhead, the errors in the r_1-direction will be neglected.

Thus, the distribution computed is projected on the reduction (i_1,p_1) plane by neglecting any of the terms with r_1 in them. Once the distribution is reduced to a two-dimensional normal, it is necessary to find its major and minor axes so that the distribution may be expressed as the product of two independent normal distributions. Before being rotated, the distribution will have the form:

$$\text{prob } (i,p) = K \exp - (i^2/V_i + \frac{2*i*p}{CV_{ip}} + p^2/V_p)$$

where

$$V_i = Tv(i_1); \quad V_p = Tv(p_1); \quad \text{and } CV_{ip} = Tcv(i_1,p_1).$$

One of the axes of the independent distribution is along the angle θ where

$$\theta = \frac{1}{2} \tan^{-1} (2*CV_{ip}/(V_p-V_i))$$

The variance along this axis is:

$$V_{maj} = (V_p * \cos(\theta)) + (V_i * \sin(\theta))$$
$$+ (2*C_{ip}*\cos(\theta) * \sin(\theta))$$

15

and along the other axis is:

$$V_{min} = (V_p * \sin(\theta) + (V_i * \cos(\theta)$$
$$- (2 * CV_{ip} * \cos(\theta) * \sin(\theta)).$$

If V_{maj} is greater than V_{min}, then θ is left alone. Otherwise, 90 deg is subtracted to generate a new θ, and V_a and V_b are interchanged. This yields the biases along the major and minor axes:

$$B_{maj} = TB(i_1) * \cos(\theta) + TB(p_1) * \sin(\theta)$$

$$B_{min} = -TB(i_1) * \sin(\theta) + TB(p_1) * \cos(\theta).$$

The five parameters θ, V_{maj}, V_{min}, B_{maj}, B_{min} fully describe the distribution of the intercept between the warhead centroid and the reduction plane. Any further analysis of the hit probabilities depends on the warhead and target characteristics.

PROBABILITY OF HIT MATHEMATICS

1.   Uniform Random Pattern

After the flight and distribution parameters of the warhead centroid have been computed, it is possible to compute the probability of hitting a relatively small target with an expanding disk of particles. The PROCM routine generates the probability of hit for the U.R. warhead.

The first problem in this analysis is to determine the radius of the disk of particle as it passes through the reduction plane. The radius of the particle disk will be determined by its expansion speed V_{exp} and its time of flight T_{int}:

$$R_{ave} = T_{int} * V_{exp}.$$

Since the disk will usually not pass through the reduction plane head on, it will leave an elliptical trace. If the angle between its velocity vector and the perpendicular to the reduction plane is Dtrg the disk will leave an elliptical trace having a major axis R_{ave} and a minor axis (R_{ave}) (cos (Dtrg)).

The program supplied by Dahlgren (Ref. 1) permits the calculation of the probability that the center of the vulnerable area is covered by the particle disk. To use this program a circular particle disk must be used, and the error distribution must be expressed as an independent bivariate normal distribution. This may be achieved by transforming the coordinater system of the reduction plane. Figure 3 shows the relevant angles and parameters in the reduction plane.

The first step in this transformation is to express the normal distribution in terms of the disk coordinates. This requires a rotation of the coordinate system of the aiming distribution by an angle of rot where rot = (Hor to aiming distribution major axis - Hor to distribution minor axis). As discussed in Cramer (Ref. 2) any linear transformation between X, Y coordinates and X', Y' coordinates may be expressed by:

17

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \overset{M}{\begin{bmatrix} a & , & b \\ c & , & d \end{bmatrix}} * \begin{bmatrix} X \\ Y \end{bmatrix} \qquad \text{i.e., } \begin{aligned} X' &= ax + by \\ Y' &= cx + dy \end{aligned}$$

For such a transformation,

$$\begin{bmatrix} \text{bias}(X') \\ \text{bias}(Y') \end{bmatrix} = M * \begin{bmatrix} \text{bias}(X) \\ \text{bias}(Y) \end{bmatrix}$$
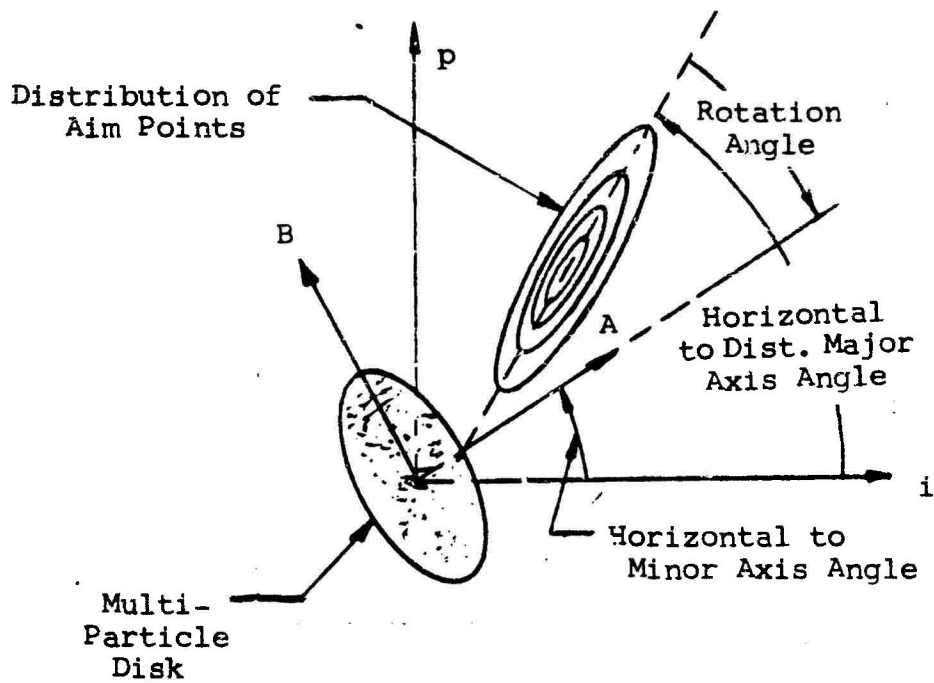

Figure 3   Reduction Plane

and

$$\begin{bmatrix} \text{var}(X') & , & \text{covar}(X'Y') \\ \text{covar}(X'Y') & , & \text{var}(Y') \end{bmatrix} = M * \begin{bmatrix} \text{var}(X) & , & \text{covar}(XY) \\ \text{covar}(XY) & , & \text{var}(Y) \end{bmatrix} * M^T$$

18

but, for $M = \begin{bmatrix} a & , & b \\ c & , & d \end{bmatrix}$

$M * \begin{bmatrix} \text{var}(X) & \text{covar}(XY) \\ \text{covar}(XY), & \text{var}(Y) \end{bmatrix} * M'^T$

$$= \begin{bmatrix} a^2 * \text{var}(X) + b^2 * \text{var}(Y) & ac * \text{var}(X) + bd * \text{var}(Y) \\ + 2ab * \text{covar}(XY) & , + (ab + bc) * \text{covar}(XY) \\ ac * \text{var}(X) + bd * \text{var}(Y) & c^2 * \text{var}(X) + d^2 * \text{var}(Y) \\ + (ad + bd) * \text{covar}(XY), & + 2cd * \text{covar}(XY) \end{bmatrix}$$

for a rotation of $<$ rot

$a = \cos(\text{rot})$ $b = +\sin(\text{rot})$ $c = -\sin(\text{rot})$
$d = \cos(\text{rot})$

Thus, in rotating to the disk system

$\text{Bias}(A) = B_{\text{Maj}} * \cos(\text{rot}) + B_{\text{Min}} * \sin(\text{rot})$
$\text{Bias}(B) = B_{\text{Maj}} * \sin(\text{rot}) + B_{\text{Min}} * \cos(\text{rot})$

$\text{Var}(A) = V_{\text{Maj}} * \cos^2(\text{rot}) + V_{\text{Min}} * \sin^2(\text{rot})$
$\text{Var}(B) = V_{\text{M}}aj * \sin^2(\text{rot}) + V_{\text{Min}} * \cos^2(\text{rot})$

$\text{Covar}(A,B) = \cos(\text{rot}) * \sin(\text{rot}) * (V_{\text{Min}} - V_{\text{Maj}})$

After the distribution has been expressed in the disk's system it is possible to "elongate" the A coordinate so that the disk may be considered to have a circular trace in the reduction plane. This is done by defining a new coordinate system A'B' so that:

$$A^t = \frac{1}{\cos(\text{Dtrg})} * A \quad \text{and} \quad B' = B,$$

using the previous notation, this transformation is obtained by setting

$$a = \frac{1}{\cos(\text{Dtrg})}, \quad b = c = 0, \quad \text{and} \quad d = 1.$$

and the distribution has new parameters as follows

$$\text{Bias}(A') = \frac{1}{\cos(\text{Dtrg})} \text{Bias}(A)$$

$$\text{Bias}(B') = \text{Bias}(B)$$

$$\text{Var}(A') = \frac{1}{\cos^2(\text{Dtrg})} \text{Var}(A)$$

$$\text{Var}(B') = \text{Var}(B)$$

$$\text{Covar}(A'B') = \frac{1}{\cos(\text{Dtrg})} \text{Covar}(A,B)$$

Next it is necessary to find the principal axes of this distribution, and express all the parameters of the distribution in terms of their coordinate system. Since this is the same transformation required in determining the major and minor axes of the original distribution, no further discussion is needed of this step.

At this point the data are in a form such that the Dahlgren (Ref. 1) program may be used to compute the probability that the center of the target is within the particle disk.

This program computes the probability that a disk of radius R_{ave} will cover a point if the center of the disk is distributed as a bivariate normal. This is given by the integral:

20

$$P(Cover) = \int_{-R_{ave}}^{+R_{ave}} dx \left[\int_{\sqrt{R_{ave}^2 - x^2}}^{\sqrt{R_{ave} + x^2}} \frac{1}{2\pi\,\sigma_x\,\sigma_y} * \exp\left[-\left[\frac{(x-\bar{x})^2}{2\sigma_x^2} + \frac{(y-\bar{y})^2}{2\sigma_y^2} \right] \right] dy \right]$$

where

σ_x and σ_y are the standard deviation of the distribution, and

$\bar{x}$ and $\bar{y}$ are its biases.

This computation results in the probability that the center of the target is covered by the particle disk.

Since the vulnerable region is assumed to be relatively small compared to the size of the particle cloud, the probability that the area is partially covered by the cloud is low relative to the probability of being completely covered or missed as discussed previously. In this case, it is possible to assume that the area is covered when the center is covered. When the area is covered by the particle cloud, it is possible that all the area is not hit if the particles of the cloud miss the area. When there are a large number of particles and the area is much smaller than the cloud, the probability that K particles hit an area within the cloud may be approximated by a Poisson distribution:

$$P(K\ hits) = \frac{\lambda^K}{K!} * e^{-\lambda}$$

where

$\lambda = \eta * \Delta A/\text{disk area}$
$\eta = \text{number of particles in disk}$
$\Delta A = \text{area of vulnerable region.}$

It should be noted that the area of the disk is not the area of the original circular disk, but of the elliptical trace in the reduction plane. The probability of no hits is $e^{-\lambda}$, therefore, the probability of one or more hits is $1 - e^{-\lambda}$.

21

It is then possible to compute the probability that an area is hit by multiplying the probability that it is covered by the cloud by the probability that if it is covered by the cloud it will be hit.

$$P(hit) = P(cover) * (1 - e^{-\lambda}).$$

At this point the required value has been computed, and the computer program is finished with the specific intercept situation, and either considers another intercept or stops.

2. Continuous Rod Mathematics

To determine some of the problems that will be encountered in studying complex warheads against complex targets it was decided to study the problem of the continuous rod warhead against a number of vulnerable regions. The warhead is an expanding solid ring that is assumed to disable the target if the ring hits any of the vulnerable regions of the target.

Mathematical Statement of Problem

In completely general terms, the probability of any event E that depends on other variables x_1, x_2, ...,x_n is defined by the equation:

$$Pr(E) = \int_{\substack{x_1,x_2,\ldots x_n \\ \text{such that } E \\ \text{occurs}}} \rho(x_1,x_2,\ldots,x_x) \; dx_1,dx_2,\ldots dx_n$$

An equivalent way of expressing this is to use the indicator function

$$\delta (***) = \begin{cases} 1 \text{ when } *** \text{ is satisfied} \\ 0 \text{ when } *** \text{ is not satisfied} \end{cases}$$

Then, it is possible to express the probability of an event E:

$$Pr(E) = \int_{\substack{\text{all} \\ x_1,x_2,\ldots,x_n}} \delta(E_{(x_1,x_2,\ldots,x_n)} \text{occurs}) \rho(x_1,x_2,\ldots,x_n) dx_1 dx_2 \cdot dx_n.$$

22

In evaluating a warhead, the event E to be con-
sidered is that of intercepting the target with the
warhead. The event of not intercepting, ~E, may also
be useful in further work. The four random variables
that affect the event are i and p, the position of
intercept of the warhead center with the reduction plane,
V_e, the velocity of the expansion of the warhead; and
T_d, the time of detonation of the warhead before it
intercepts the reduction plane. These last two vari-
ables, V_e and T_d, may be combined to give the size
of the ring, S_r, at intercept:

$$S_r = (V_e) \cdot (T_d)$$
$$= (V_e) * (Vw/Dd)$$

where Vw is the velocity of the warhead relative to the
target and Dd is the distance from warhead to reduction
plane at detonation. Then:

$$Pr(hit) = \int_{i,p,s} \delta(hit/i,p,s) \; \rho(i,p,s) \; di \quad dp \quad ds \qquad (1)$$

The $\rho(i,p,s)$ term depends on the distribution of errors
in aiming and detonating the warhead. If the distribu-
tion of ring radii is independent of the center location
and $\rho(i,p)$ is a bivariate normal, then, by proper def-
inition of the (i,p) direction, the distribution may be
broken into three independent terms:

$$\rho(i,p,s) = \rho(i) * \rho(p) * \rho(s). \qquad (2)$$

The function $\delta(hit/i,p,s)$ is somewhat more difficult
to evaluate. This term depends on the shape of the ring
intersection with the reduction plane and on the vulner-
able regions. The actual shape of the ring is an annulus
of relatively small width, but it may not be perpendicu-
lar to the relative flight path of the warhead, thus,
may intersect the reduction plane in an ellipse.

Consider the situation shown in Figure 4. If the
ring is at an angle α with the reduction plane and
the radius is s, then the intersection of the ring
with the (X,Y) plane is an ellipse given by

$$x^2/\cos(\alpha) + y^2 = s^2.$$

23
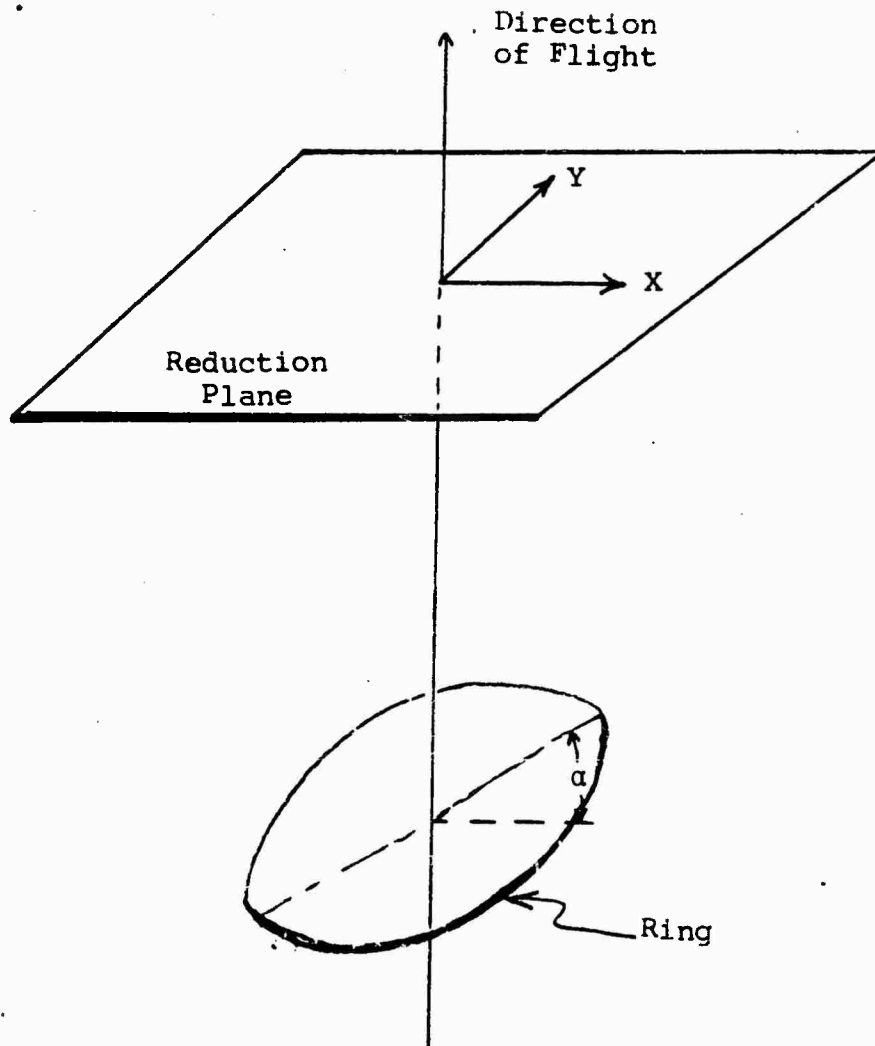
Figure 4   Intercept of ring and reduction plane

The indicator function for a set of vulnerable areas
is the Boolean sum of indicator functions for each vul-
nerable region. This function is one if any vulnerable
region is intercepted by the ring, and zero otherwise.
Let us consider a relatively simple case of one vulner-
able region where the ring intersection and the vulner-
able area are both circles. In this case the volume
where the interception occurs is a truncated hollow cone.

24

If the vulnerable region is centered at the point (0,0) with radius R, interception occurs if i,p, and s satisfy both the equations (see Figure 5).

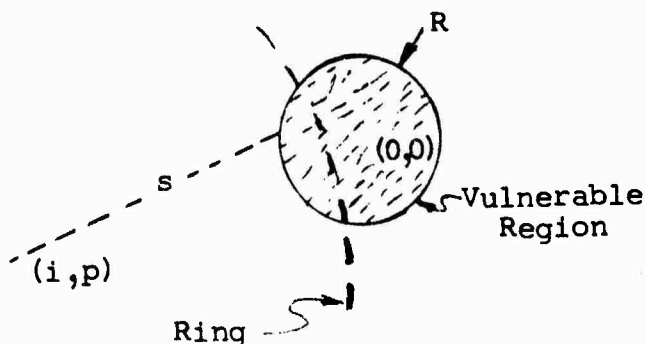$$i^2 + p^2 \geq s^2 - R^2 \text{ and } i^2 + p^2 \leq s^2 + R^2$$

Figure 5 Intercept of circular ring
and circular vulnerable region

When more than one vulnerable region is involved, the intercept occurs when the i, p, and s values satisfy one or more sets of equations of the type shown.

Two factors complicate this analysis in the real problem: (1) First, the ring intersection is an ellipse rather than a circle, and (2) more than one vulnerable area is involved. The first problem means that instead of being cones with circular cross sections, the boundaries of the intersection volumes have approximately elliptical cross sections. The second, and really more difficult problem, is that a number of these cone-shaped areas must be considered. The volumes that must be integrated over are thus relatively complex, and would be difficult to characterize, even in the simple case where the ring intersection is a circle.

25

A Numerical Technique for Evaluating
a Continuous Rod

In the case of where the size s is independently distributed relative to the intersection point i,p, the probability of hit by a warhead may be given, in general, by combining evaluations (1) and (2) to obtain the equation:

$$\text{Pr(hit)} = \int_s \left(\int_i \int_p \delta(\text{hit}/i,p,s)\ \rho(i,p)\ di\ dp \right) \rho(s)\ ds$$

To analyze this by numerical methods it is necessary to approximate the integrals by sums:

$$\text{Pr(hit)} \cong \sum_{s=s_o-k_1\Delta s}^{s=s_o+k_1\Delta s} \left(\sum_{i=i_c-k_2\Delta i}^{i_o+k_2\Delta i} \sum_{p=p_o-k_3\Delta p}^{p_o+k_3\Delta p} \delta(\text{hit}/i,p,s)\rho(i,p)\Delta i\Delta p \right)$$

$$* \rho(s)\Delta s$$

The indicator function $\delta(s,i,p)$ specifies which combinations of $s,i,$ and p result in a hit. This function is independent of the distribution of these parameters. It was decided to evaluate this indicator function first since it may then be used to evaluate a number of different aim-point, and ring-size distributions. The program will determine the indicator function relatively early, and later the program will sum this function over various distributions.

This program may be divided into major processing stages:

I. Computation of $\delta(s,i,p)$ for a set of grid points in the (i,p) plane.

II. Computation of $\text{Pr(hit}/s) = \sum_i \sum_p \delta(s,i,p)\ \rho(i,p)\Delta i\Delta p$
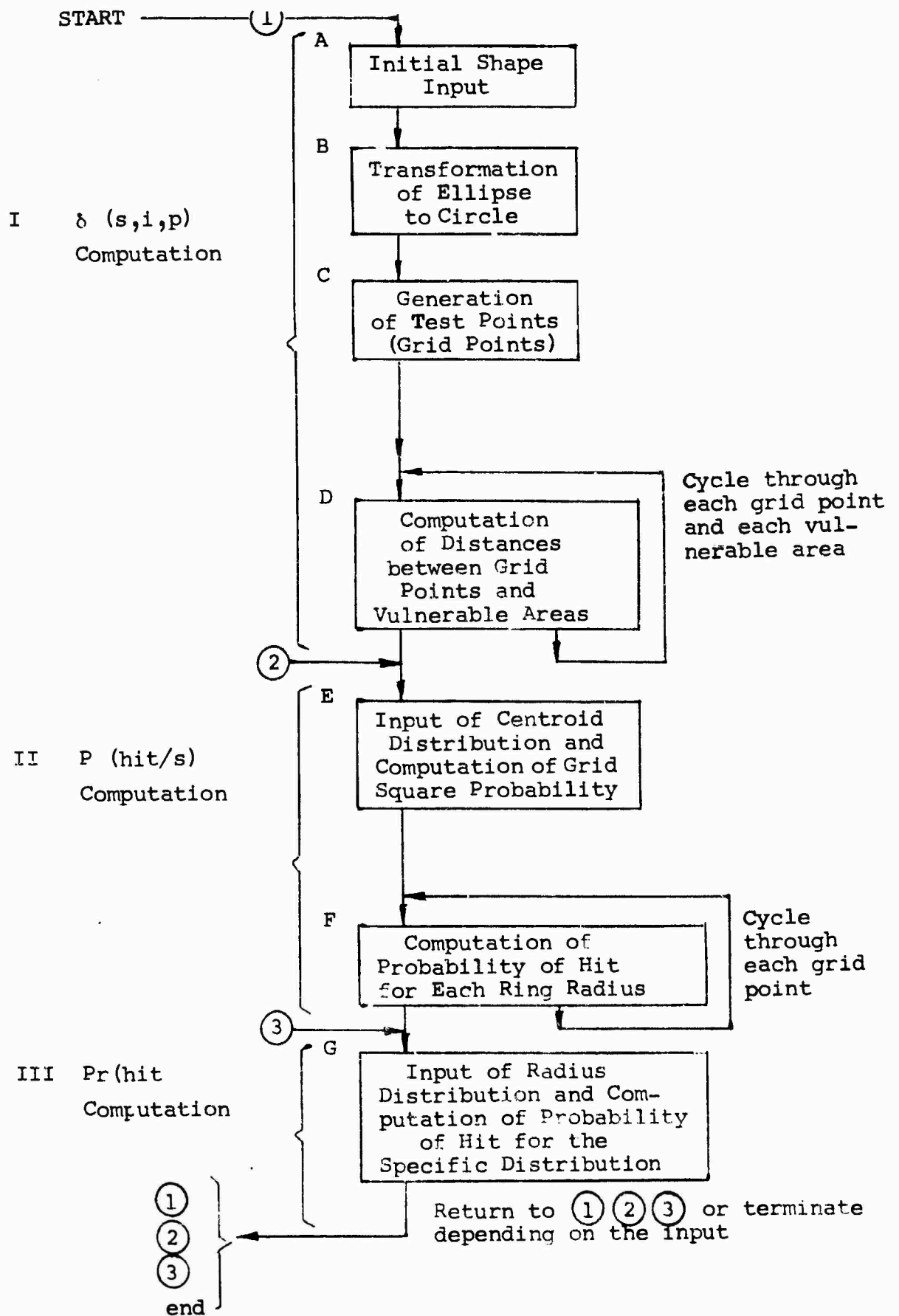for any specific distribution of grid points.

Figure 6   Flow chart of continuous rod warhead program

III. Computation of Pr(hit) for any specific
distribution of ring sizes.

These three major processing stages are then
divided into a total of seven stages. A flowchart of
these stages is given in Figure 6.

Stage A: Initial Input Data (Shape Data). Some input
data are required before any processing can be done.
These data are a description of the vulr·rable areas,
the shape of the ring warhead intersection with the
reduction plane, and estimation as to the size of the
various distribut'.ons, and the accuracy desired in the
final answer.

The description of the vulnerable areas could be
given in a number of ways. To simplify some of the
computations, it was decided to describe the vulnerable
areas as a limited number of polygons each with a
limited number of sides. These data are stored in the
following way:

$$\text{iar} = \text{number of areas}$$
$$\text{ico(i)} = \text{number of corners or sides of each area}$$
$$\text{cx(i,j),cy(i,j)} = \text{coordinates of the } j^{th} \text{ corner}$$
$$\text{of the } i^{th} \text{ shape.}$$

The shape of the ring intersection is given by its
orientation and eccentricity. These will be given by
an angle and a ratio of the major to minor axis:

$$\text{rt} = \text{angle of major axis with X axis}$$
$$\text{re} = \text{major to minor axis ratio.}$$

The size of the aim points distribution and the
ringò size distribution must be estimated to determine
the grid size that must be used in later stages.

$$\text{sc} = \text{The average standard deviation of the centroid distribution}$$
$$= 1/2 \left\{ \begin{array}{l} \text{maximum standard deviation} \\ + \text{ minimum standard deviation} \end{array} \right\}$$
$$\text{sy} = \text{coordinates of the centroid aim point}$$
$$\text{sr} = \text{the standard deviation of the ring radius}$$
$$\text{r zero} = \text{average ring radius.}$$

28

These values are estimated so that the proper grid size is used in each case. Various actual distributions will be input later. If these values are properly estimated, then one calculation will suffice for a number of different distributions. The accuracy of a specific calculation will depend on the grid size. To reduce the computation time, it is desirable to be able to control this size. This is done by specifying the number of divisions of the grid in each direction in the reduction plane:

ngd = number of grid divisions in each direction.

Stage B: Ellipse to Circle Transformation. Since it is easier to deal with a circular ring, the first translation is to distort the X,Y plane into an (i,p) plane in which the ring intersection is a circle. This is represented by a rotation by $-\theta$, A, a stretch of Y axis by re, B, and a rotation by $+\theta$, C.

$$\begin{bmatrix} i \\ p \end{bmatrix} = \overset{C}{\begin{bmatrix} \cos\theta, & \sin\theta \\ -\sin\theta, & \cos\theta \end{bmatrix}} \cdot \overset{B}{\begin{bmatrix} 1 & 0 \\ 0 & e \end{bmatrix}} \cdot \overset{A}{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}} \cdot \begin{bmatrix} X \\ Y \end{bmatrix}$$

θ = rr   and   e = re

This transformation should be applied to the coordinates of the corners as the vulnerable areas and the bias of the centroid distribution to obtain the new values in the (i,p) plane. To save storage the transformed values will be stored in the original locations, and be denoted by the same symbols. This transformation should be saved, since other values must be expressed in the (i,p) system.

Stage C: Grid Generation. It is necessary to generate a set of grid points, each to be used as a location for the center of the ring. The separation of these points will depend on the number of grid divisions (ngd) desired, and on the average standard deviation of the aim point sc. One technique for choosing the separation is to let the probability of a tail of the normal be equal to an average probability in any specific slot. The average probability of being in any slot is approximately 1/ngd. A rough estimate of the probability of being in a tail is

29

$$\frac{1}{\sqrt{2\pi}} \ \frac{1}{x} \ \exp \ (-x^2/2)$$

where $x = \text{cutoff}/\sigma$. A relatively simple table can be stored for the cutoff point, cut(ngd), as a function of ngd. Then the separation, sep, is given by

$$\text{sep} = \text{sc} \ . \ \text{cut(ngd)}/\text{ngd}.$$

The grid points will then be selected by:

$$x_{gp} = sx \pm k_1 \ * \ \text{sep}$$

$$y_{gp} = sx \pm k_2 \ * \ \text{sep}$$

with:

$$0 \le k_1 \le \text{ngd}/2$$

$$0 \le k_2 \le \text{ngd}/2.$$

Distance Determination. For each grid point, a set of radii that the ring can have and intercept a vulnerable area will be computed. Let us consider one area with corners (in the (i,p) plane), cx_i, cy_i, and a grid point at $g = (gx, gy)$. The distance to a corner is given by:

$$\text{dis}_i = ((cx_i - gx)^2 + (cy_i - gy)^2)^{1/2}$$

The maximum of these distances is the maximum radius the ring may have and still intercept the area. This will be called r_{max}. Similarly, a minimum distance corner, tentatively called r_{min}, exists. Two further considerations must be considered in determining r_{min}. First, it is possible that the grid point is inside the vulnerable area. In this case, $r_{min} = 0$. Second, the ring may pass through a side rather than the nearest corner and, in this case, r_{min} is less than that based on the corners.

To test whether the point is inside, we use the Jortan curve theorem. Consider the infinite half-line drawn from the grid point (gx, gy) to the "infinite" point (gx, ∞). A line between two corners (cx_i, cy_i)

and (cx_{i+1}, cy_{i+1}) is crossed by the infinite half-line if, and only if,

$$(cx_i-gx) * (cx_{i+1}-gx) < 0$$

and

$$gy < (cy_{i+1}-cy_i) * \left[\frac{cx_2 - gx}{cx_2 - cx_1} \right].$$

A point is inside the area if the infinite half-line crosses such boundaries an odd number of times.

If a corner of the area is on the half-line, then an intersection of the half-line with the boundary occurs when the y coordinate of the corner is greater than gy and the two adjacent corners are not on the same side of the half-lines. Thus, it is possible to determine when a grid point is inside an area.

The second consideration occurs when the grid point is outside the area. It is possible that the nearest point on the boundary of the area is not one of the corners, but is a side.

A crude way of considering this complication is to examine each set of sequential corners, determine whether the point of closest approach of the line to the point lies between the corners, and if it does, and if the distance is less than r_{min} replace by this distance.

Consider two points, a and b. The line between these is given by the following parametric equation:

$$Lx = ax + (bx-ax) * Q$$
$$Ly = ay + (by-ay) * Q$$
(3)

The distance between a line point and the grid point is given by the equation:

$$dis\ (g,L) = (gx-Lx)^2, (gy-Ly)^2)^{1/2}$$

$$= (gx-ax + (bx-ax)Q)^2$$

$$+ (gy-ay + (by-ay)Q)^2)^{1/2}$$

31

To find the minimum point the distance squared may
be differentiated with respect to Q.

$$\frac{\partial \text{dis}^2}{\partial Q} = (gx-(ax+(bx-ax)Q) \cdot (Bx-ax)$$

$$+ (gy-ay+(by-ay)Q) \cdot (by-ay)$$

$$= 0 \text{ if and only if}$$

$$Q = \frac{(bx-ax) \cdot (gx-ax) + (by-ay) \cdot (gy-ay)}{(bx-ax)^2 + (by-ay)^2}$$

If $Q \geq 1$ or $Q \leq 0$ this line need not be con-
sidered since the closest point is not between a and
b. If $0 < Q < 1$ then Q may be substituted in equa-
tion (3) to determine a minimum distance for that line.

By the preceding operations it is possible to
determine both a minimum and maximum radius such that
a ring centered at the point (gx, gy) would intersect
the vulnerable area if and only if its radius r is
between r_{min} and r_{max}. By examining each area in
turn, it is possible to obtain the total set of these
radii. These radii may then be ordered and any over-
laps may be combined. Overlaps occur when the maximum
radius that intercepts one area is larger than the
minimum radius that intercepts the next further area.
In this case, a radii interval from the minimum radius
of the closest area to the maximum radius of the fur-
ther area can be obtained.

Using these techniques, a set of radii intervals
for which intercept occurs may be generated for each
grid point. These will then be temporarily stored
along with the grid point coordinates on tape for fur-
ther processing in stage F.

Stage E: Centroid Distribution Input. The input in-
formation about the centroid location used up to this
point in the program was only used to generate the
grid points. These should have been chosen so that
the data that have been stored on tape may be used for
a large number of specific centroid distributions.
To further process the data, a specific distribution

32

must be input. A centroid distribution is determined
by its bias, its two standard deviations, and by the
direction of the maximum standard deviation with re-
spect to the x axis.

    bcx = bias of centroid in x direction
    bcy = bias of centroid in y direction
    scu = standard deviation of centroid in
          major axis direction
    scv = standard deviation of centroid in
          minor axis direction
    ag = angle between x axis and major
          axis of distribution.

Since the test points have X and Y coordinates it
is useful to determine the variance along the X axis,
along the Y axis, and the covariance between the two
axes:

$$Vx = \cos(ag)^2 * scu^2 + \sin(ag)^2 * scv^2$$
$$Vy = \sin(ag)^2 * scu^2 + \cos(ag)^2 * scv^2$$
$$CVyx = \cos(ag) . \sin(ag) * (scu-scv)$$

The probability density at any test grid point is
then determined by the normal density equation:

$$den(gx,gy) = \frac{1}{2\pi\, Vx.Vy} * \exp\left(-\left(\frac{(gx-bcx)^2}{2*Vx} + \frac{(gy-bxy)^2}{2*Vy}\right.\right.$$

$$\left.\left. + \frac{(gx-bcx)(gy-bcy)}{CVxy}\right)\right).$$

The area represented by each grid square is sep^2, thus
the approximate probability of being within the area
represented by a grid point is

$$P(gx,gy) = sep^2 * den(gx,gy). \tag{5}$$

Stage F: Probability of Intercept Given any Radius.
By using the radii intervals already computed, and the
probability that a shot is within any grid square, it
is possible to generate the probability that an inter-
cept occurs for any specified radius. For further com-
putation we will produce a table of probability of hit
for discrete radii. The table will contain the proba-
bility for ngd radii intervals with radii

33

$$r = r \text{ zero} \pm (K \cdot cut(ngd) * sr/ngd)$$

$$\Delta r_j = (cut(ngd) * sr/ngd)$$

The information stored by stage D can be used to generate the table of $Pi(rj)$, the probability of intercepting a vulnerable area with a ring whose radius is between rj and $rj + \Delta rj$.

The stored information contains gx and gy so that the probability of being within the area represented by the grid point, $P(gx,gy)$, can be calculated by equation (5). After the $Pi(rj)$ table has been initially zeroed, the $P(gx,gy)$ should be added to each of the cells of $Pi(rj)$ where rj is such that intercept occurs. In addition, estimates will be made of the probability that a grid point will have interfering radii outside the range of those represented in the matrix, or that the intercept point is entirely outside the grid considered.

This stage produces the second set of intermediate data, the probability of intercept for each radius. For design and evaluation of warheads these data should be output.

Stage G: Intercept Probability for a Specific Radii Distribution. The distribution of ring radii will be specified by an average radius, r zero, and a standard deviation, sr. The probability of intercept may be obtained by multiplying the probability of intercept given a specific radius interval by the probability of that specific radius interval occurring given by the normal function:

$$Pr(r_i) = \frac{1}{\sqrt{2\pi}\ sr}\ \exp\left(-\left(\frac{(rl - r\ zero)^2}{2 * sr^2}\right)\right)$$

and summing over all radii.

As before, an indication should be given as to the probability that a radius is less than the smallest or greater than the largest radius considered.

34

The answers generated from this state are the final result of the program and are output. After this is done, the program attempts to read more data. The data may cause the program to return to Stage A, Stage E, or Stage G, depending on the amount of new data available. If no new data are read, the program terminates.

A feasibility study was made to examine the utility of this approach for analyzing complex warheads and complex vulnerable regions. This study indicates that this approach is practical, but obviously slower than a technique which leads to an analytic or computational solution. Unfortunately, it also indicates that the analysis of complex situations will probably require the type of iterative solution outlined.

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)	2a REPORT SECURITY CLASSIFICATION
IIT RESEARCH INSTITUTE	Unclassified
10 West 35th Street, Chicago, Illinois 60616	2b GROUP

3 REPORT TITLE

SPACE ENCOUNTER MODELING STUDY
Volume II Mathematical Models

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)
Final Report 28 Feb 1964 – 28 Feb 1965

5 AUTHOR(S) (Last name, first name, initial)

Liveright, Michael

6 REPORT DATE	7a TOTAL NO OF PAGES	7b NO OF REFS
June 1966	47	2

8a CONTRACT OR GRANT NO	9a ORIGINATOR'S REPORT NUMBER(S)
AF08(635)-4203	K6064FR
b PROJECT NO	
c	9b OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d	ATL-TR-66-50 Vol. II

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13 ABSTRACT Studies conducted to provide the Air Force with mathematical models of probable space tactical situations are described in this four-volume report. The need for these models was established in order that the Air Force have means of guiding the logical and timely development and evaluation of nonnuclear space warhead concepts.

The model described can determine:

• Warhead fuzing and/or firing time (projected warheads) for a constant velocity intercept with a given miss-distance or miss-distance distribution.

• Warhead-target intercept error dispersion as a function of warhead fuzing and/or firing time for any combination of intercept parameter errors.

• Probability of hit of a given size vulnerable volume by a uniform random pattern-type warhead.

• Over-all probability of hit that is optimum for a given miss-distance distribution.

• Over-all probability of hit using a fuzing logic based upon a given value of some intercept parameters (i.e., line-of-sight range, line-of-sight bearing rate).

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13. **ABSTRACT:** Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

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